

# About Stable Models of Non-stratified Logic Programs (Abstract)

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In this paper we analyze the relationship between cyclic definitions and consistency in Gelfond-Lifschitz's answer sets semantics. As it is well-known, under the answer set semantics a theory may have no answer sets, since the corresponding general logic program may have no stable models [GelLif88].

This paper introduces a fundamental result, which is very relevant for Answer Set programming [MarTru99], and planning [Lif99]. For the first time, the class of logic programs for which a stable model exists is given a syntactic characterization (the result extends naturally to answer sets semantics). On the one hand, this condition is of theoretical interest, since it is the first one ever defined, since the introduction of the stable model semantics in [GelLif88]. On the other hand, it has a practical importance both for defining new algorithms for checking consistency and computing answer sets, and for improving the existing systems.

The methodology of this paper is to introduce a new *canonical form* to which any logic program can be reduced to. The technical result is then given in terms of programs in canonical form (canonical programs), without loss of generality. Canonical programs focus the attention on cyclic dependencies. Rules are kept short, so as to make the syntactic analysis of the program easier. The stable models of any general logic program coincide (up to the language) to those of the corresponding canonical program. Canonical programs are WF-irreducible, i.e. all the atoms involved have truth value "undefined" under the Well-founded semantics. In fact, these are exactly the atoms which are of interest for finding the stable models, as proved in [Cos95].

While a detailed analysis of the cost involved in reducing programs to their canonical form is still to be carried out, intuition suggests it is tractable and, moreover, it is important to notice that Answer Set programming encodings of

planning found in literature *are often in quasi-canonical* form. Alternatively, all definitions and results presented in this paper might be rephrased for general programs, at the expense of a lot of additional details. This means that reduction to canonical form is not strictly required for an implementation.

The result is based on identifying the cycles contained in the program, and on defining the *cycle graph*. Each vertex of this graph corresponds to one cycle, and each edge corresponds to one *handle*, which is a literal that, occurring in both cycles, actually determines a connection between them. In fact, the truth value of the handle in the cycle where it appears as the head of a rule, influences the truth value of the atoms of the cycle(s) where it occurs in the body.

We can therefore introduce the concept of a *handle path*, connecting different cycles. Cycles can be even, if they consist of an even number of rules, or vice versa they can be odd. Problems for consistency, as it is well-known, originate in the odd cycles. If, for each odd cycle: (i) we can find a handle path coming from an even cycle (*even handle path*), and (ii) this handle path is maximal, i.e. cannot be continued to another even cycle (this guarantees that all possible relevant dependencies are being considered), and moreover the handle path for different odd cycles are non-conflicting, in the sense that they do not require opposite truth value for the same value, then the existence of stable model is guaranteed, as stated in the following:

**Theorem 1** *A program  $\Pi$  has stable models if and only if, given the set  $OC_1, \dots, OC_r$  of the odd cycles of  $C$ , there exists a corresponding set  $HP_1, \dots, HP_r$  of maximal even non-conflicting handle paths, where  $HP_i$  is a handle path for  $OC_i$ .*

## References

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