The Big-O Problem is Decidable for Max-Plus Automata (PSPACE-complete)

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In this room

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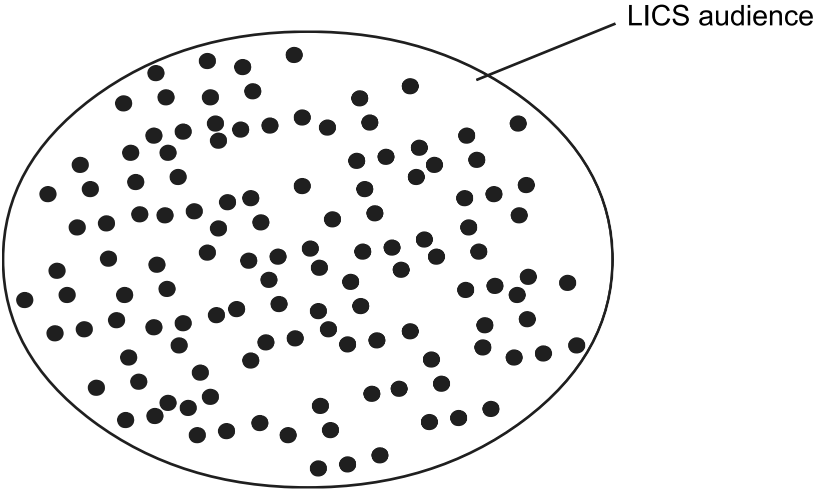

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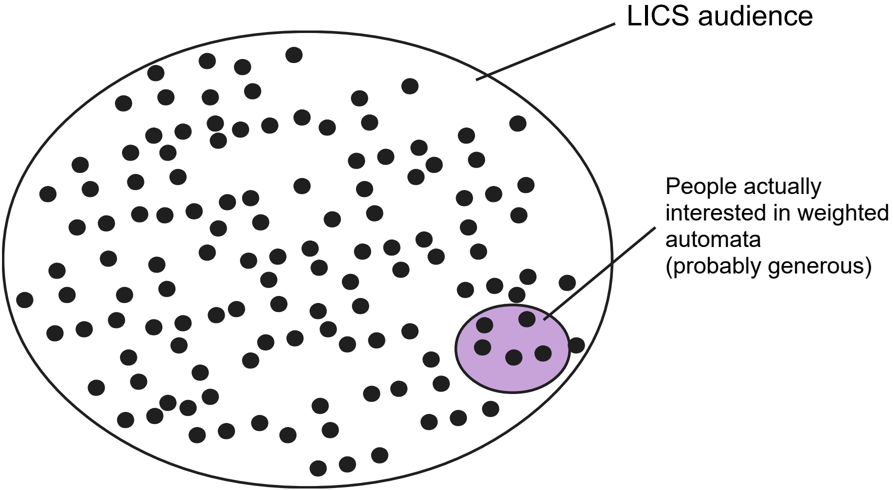
LICS audience
People actually
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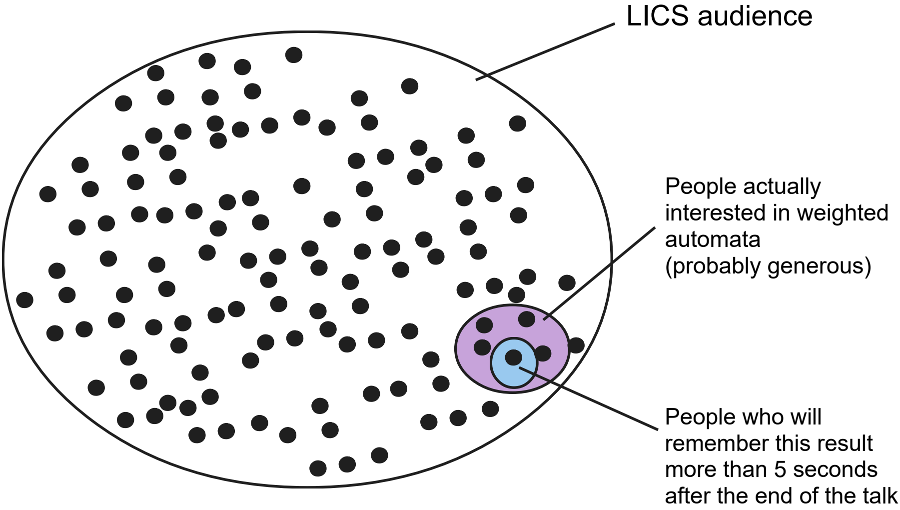
LICS audience
People actually interested in weighted automata (probably generous)
People who will remember this result more than 5 seconds after the end of the talk

Game plan

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Aim: Decide some property P on an abstract model M

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(easier said than done...)

The property and the model

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Max-Plus Automata

Extension of Boolean automaton with non-negative integers on transitions, combined using operations max and sum.

$$\Sigma^* \to \mathbb{N} \cup \{-\infty\}$$

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Given two max-plus automata computing functions f and g, is f big-O of g?

There exists C such that for all $w \in \Sigma^*$, $f(w) \leq Cg(w) + C$

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$\longrightarrow$ Undecidable: big-O for plus-times [Chistikov, Kiefer, Murawski, P.]
$\longrightarrow$ Decidable: approximation, min-plus
[Hashiguchi, Leung, Simon, Colcombet-D.-Zuleger]

One insight: turning infinite into finite

What happens when you repeat a word in an automaton?

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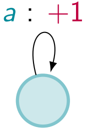

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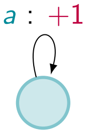

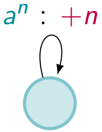

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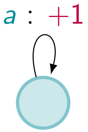

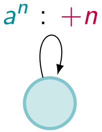

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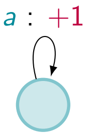

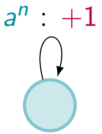

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Problem!

f: $w \mapsto |w|$

g: $w \mapsto$ max(number of b, number of consecutive a)

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f: $w \mapsto |w|$

g: $w \mapsto \max(\text{number of } b, \text{number of consecutive } a)$

$(a^n b)^n \mapsto n^2$

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f	g
$w \mapsto \|w\|$	$w \mapsto \max(\text{number of } b, \text{number of consecutive } a)$
$(a^n b)^n \mapsto n^2$	$(a^n b)^n \mapsto n$

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Problem!

f	g
$w \mapsto \lvert w \rvert$	$w \mapsto \max(\text{number of } b, \text{number of consecutive } a)$
$(a^n b)^n \mapsto n^2$	$(a^n b)^n \mapsto n$

$(a^\sharp b)^\sharp = (\infty, \infty)$

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$(a^\sharp b)^\sharp = (\infty, \infty)$ but $(a^\sharp b)^\flat = (\infty, 1)$

$\longrightarrow$ New flattening operation $\flat$

The End (or just the beginning)

Theorem [D., Purser]

The big-O problem is PSPACE-complete for max-plus automata.