Structure-Aware Lower Bounds and Broadening the Horizon of Tractability for QBF

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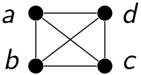

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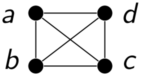
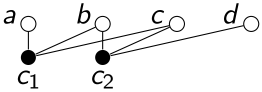
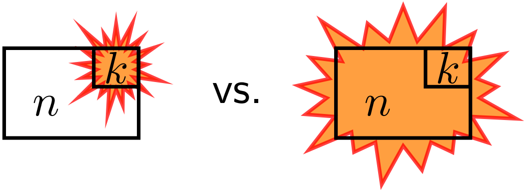


Incidence graph I_φ



Motivation

- Treewidth k [Robertson & Seymour 83, Bertele & Brioschi 69]
  - renders large variety of NP-hard problems *fixed-parameter tractable (FPT)*



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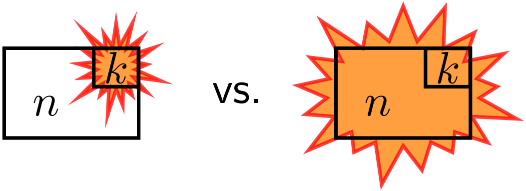

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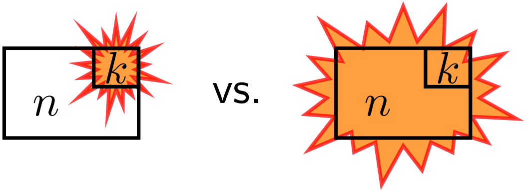

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⇝ Frameworks & Solvers: [Langer et al. 12], [Bliem et al. 16], [Bannach & Berndt 19], [Charwat & Woltran 19], [F et al. 22],...

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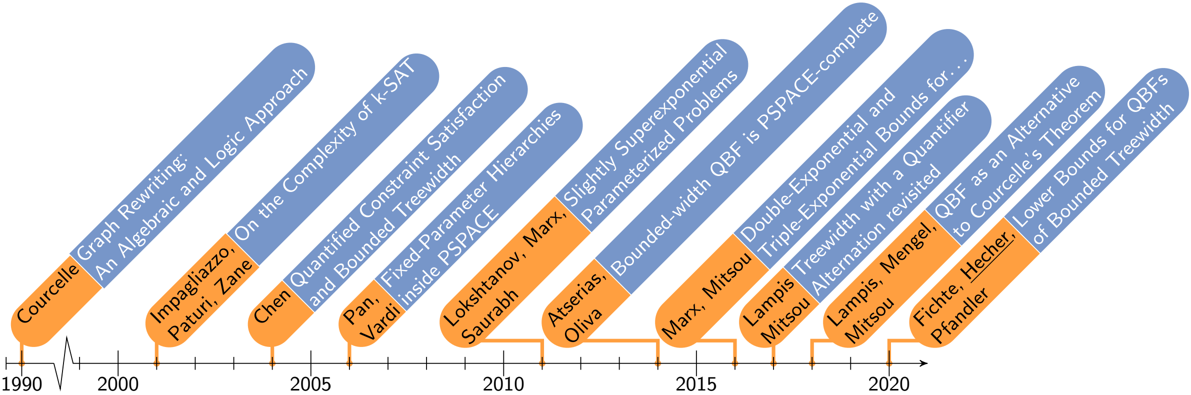
Problem	Bounds $f(k)$ Upper	Lower (ETH)	Complexity
SAT	$2^{\mathcal{O}(k)}$	$2^{o(k)}$	NP
QSAT_ℓ	$\mathrm{tower}(\ell, \mathcal{O}(k))$	$\mathrm{tower}(\ell, o(k))$	Σ_ℓ^{P} / Π_ℓ^{P}

Excerpt of Results



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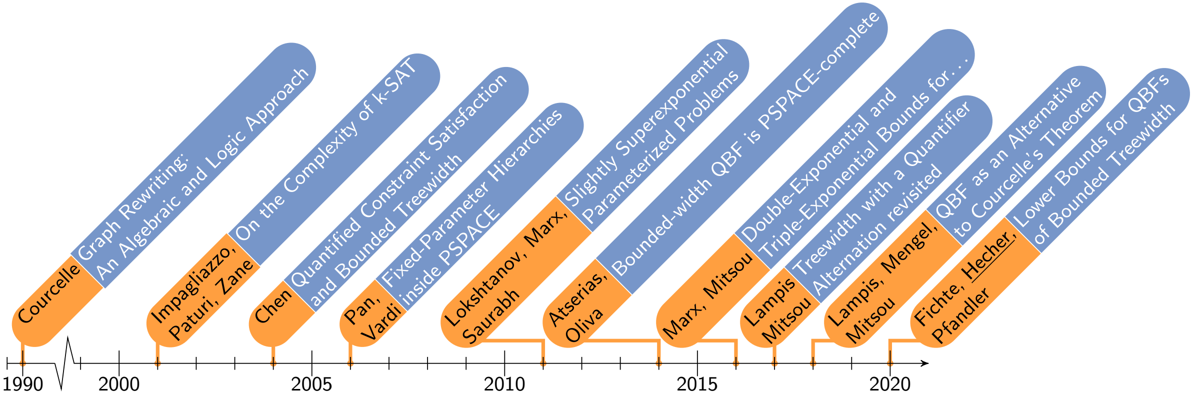

⇝ For QSAT_ℓ and vertex cover number s: $f(s) = 2^{\mathcal{O}(s^3)}$ [Lampis & Mitsou 17]

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Unlikely for known parameters p (incidence graph)

Contributions

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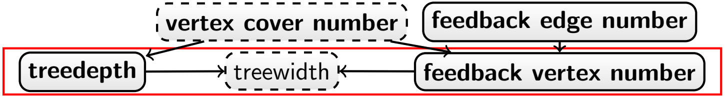

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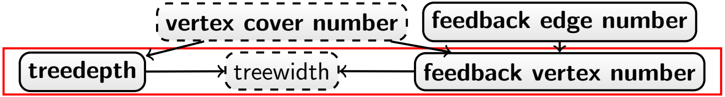

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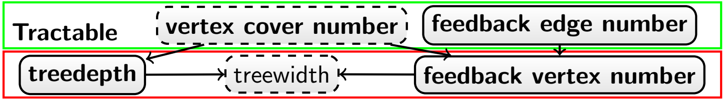

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Core Idea

Standard approach	Our approach
(SAT, p)	(SAT, p)
$\downarrow$	$\downarrow_{SAW}$
$(\mathrm{QSAT}_2, \log(n))$	$(\mathrm{QSAT}_2, \log(p))$

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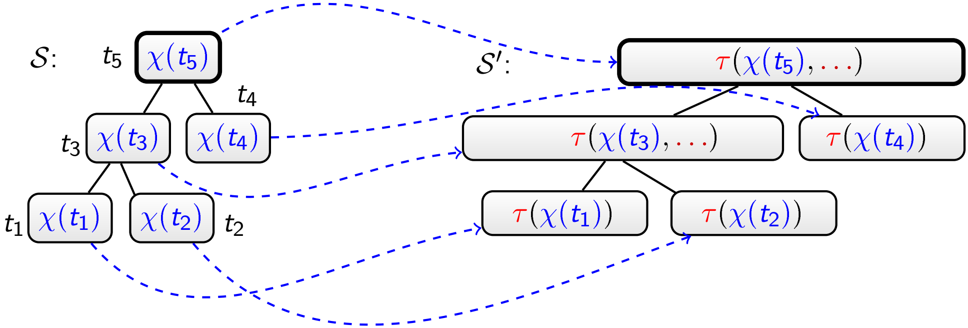

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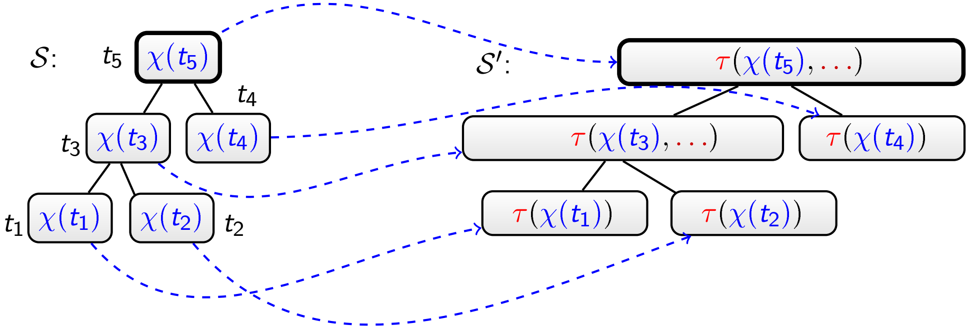
Structure-Aware (SAW) Reductions



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$\rightsquigarrow$ $\mathcal{S}'$ functionally depends on $\mathcal{S}$

Outline for (Sparse) Feedback Vertex Number

Example 2

Given: QBF with 3-CNF matrix φ, feedback vertex set S of P_φ

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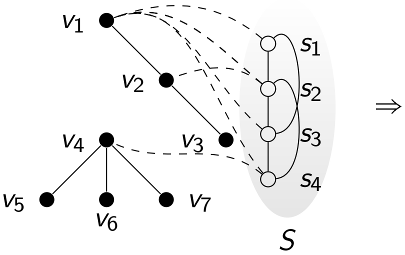

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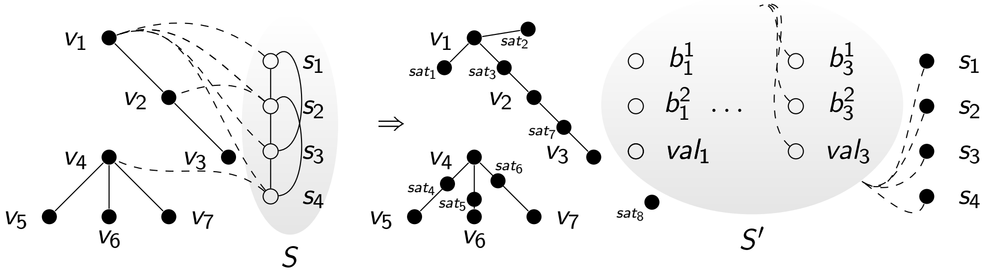

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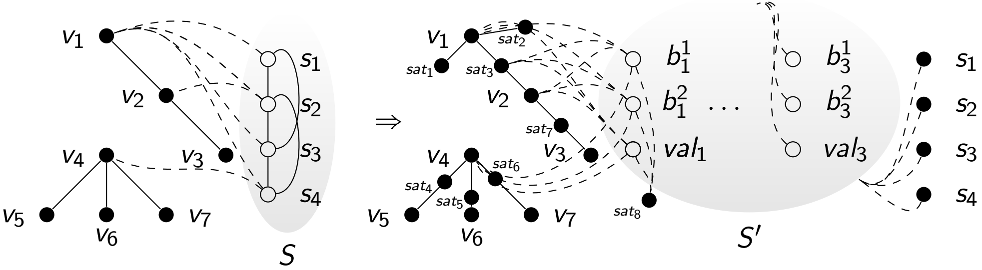

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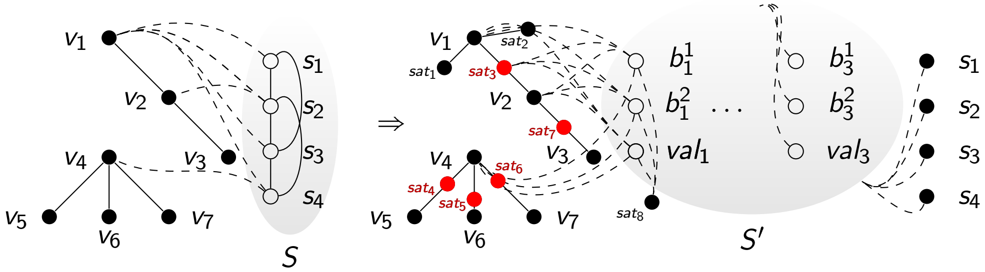

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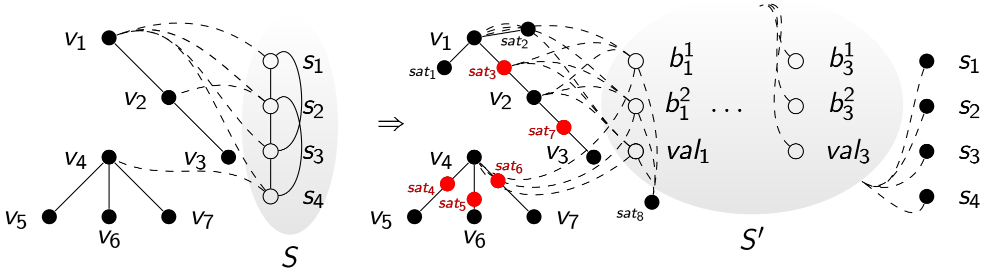

⇝ Additional auxiliary variables under inner-most universal quantifier block

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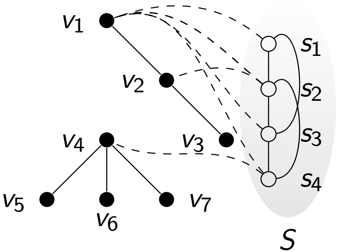

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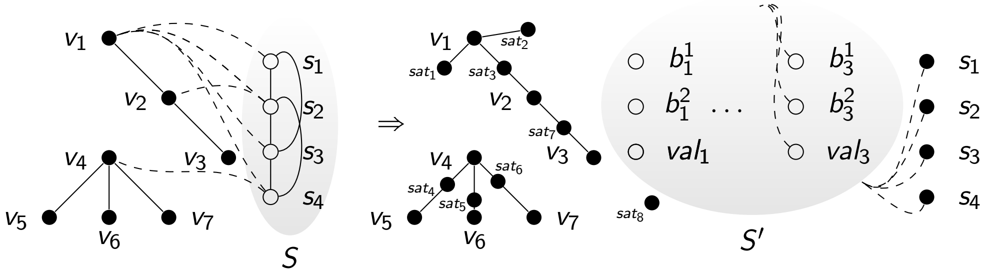

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Given a QBF of the form $Q = Q_1 V_1.Q_2 V_2.\cdots Q_\ell V_\ell.\varphi$ where φ is in 3-CNF (assuming $Q_\ell{=}\exists$). Under ETH, Q cannot be decided in time $\mathsf{tower}(\ell, o(s)){\cdot}poly(\|\varphi\|)$, where s is the (sparse) feedback vertex number of the incidence graph I_φ.

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$\rightsquigarrow$ The reduction also works for the incidence graph over 3-CNFs

Theorem 5 (QBF lower bound)

Given a QBF of the form $Q = Q_1 V_1 . Q_2 V_2 . \cdots Q_\ell V_\ell . \varphi$ where φ is in 3-CNF (assuming $Q_\ell = \exists$). Under ETH, Q cannot be decided in time $\mathsf{tower}(\ell, o(s)) \cdot poly(\|\varphi\|)$, where s is the (sparse) feedback vertex number of the incidence graph I_φ.

Parameter p	NF Primal (NF Inc.)	Bounds $f(k)$	
		Upper	Lower (ETH)
(sparse) FVN	3,1-CDNF (3-CNF)	$\mathsf{tower}(\ell, \mathcal{O}(p))$	$\mathsf{tower}(\ell, o(p))$
Dist. half-ladder	3,1-CDNF (3-CNF)	$\mathsf{tower}(\ell, \mathcal{O}(p))$	$\mathsf{tower}(\ell, o(p))$
treedepth	3,1-CDNF (3-CNF)	$\mathsf{tower}(\ell, \mathcal{O}(p))$	$\mathsf{tower}(\ell, o(p-\ell))$, $\mathsf{tower}(\ell - \log^*(p) - 2, o(p))$

Conclusion and Open Questions

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Future Work

- Additionally consider quantifier dependency (schemes)
- Deeper study of bounds under SETH

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Future Work

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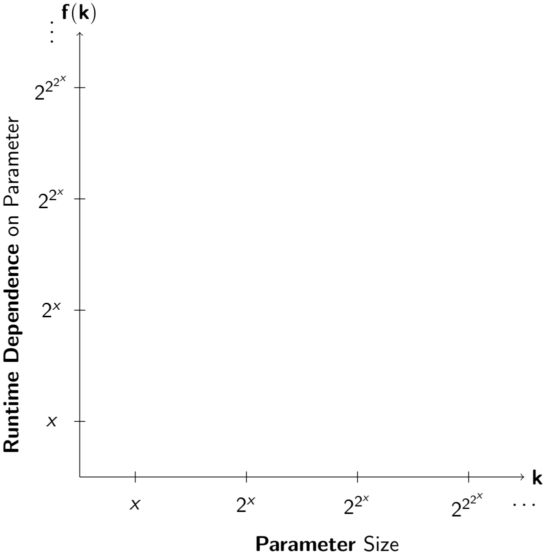
- Deeper study of bounds under SETH
- What about the primal graph? Distance to disjoint paths?

Trade-off between Time and Structure



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